Translations and Vectors

01 Introduction

When you want to ask for a place, you ask the distance to the place or the direction to it???

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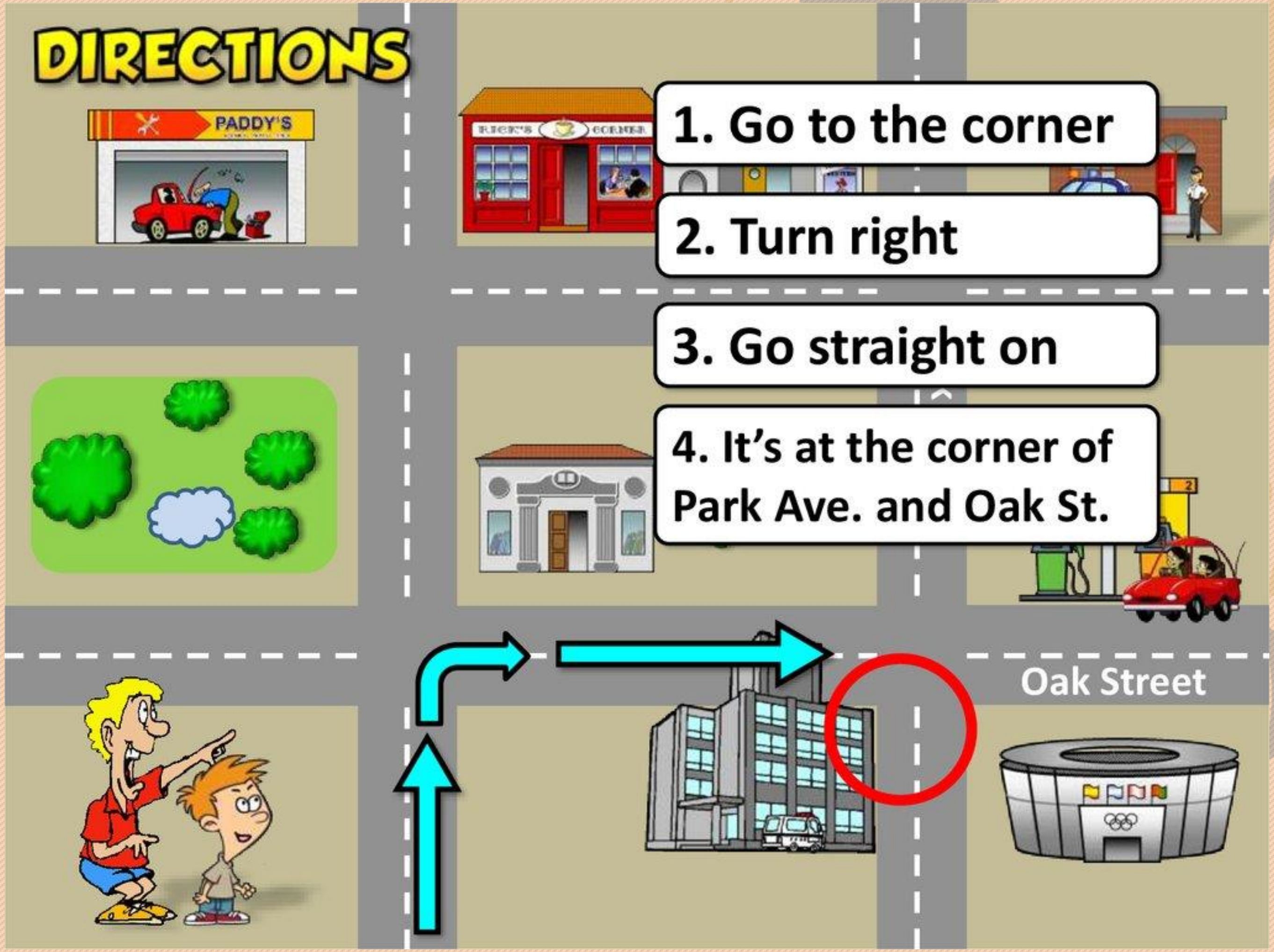
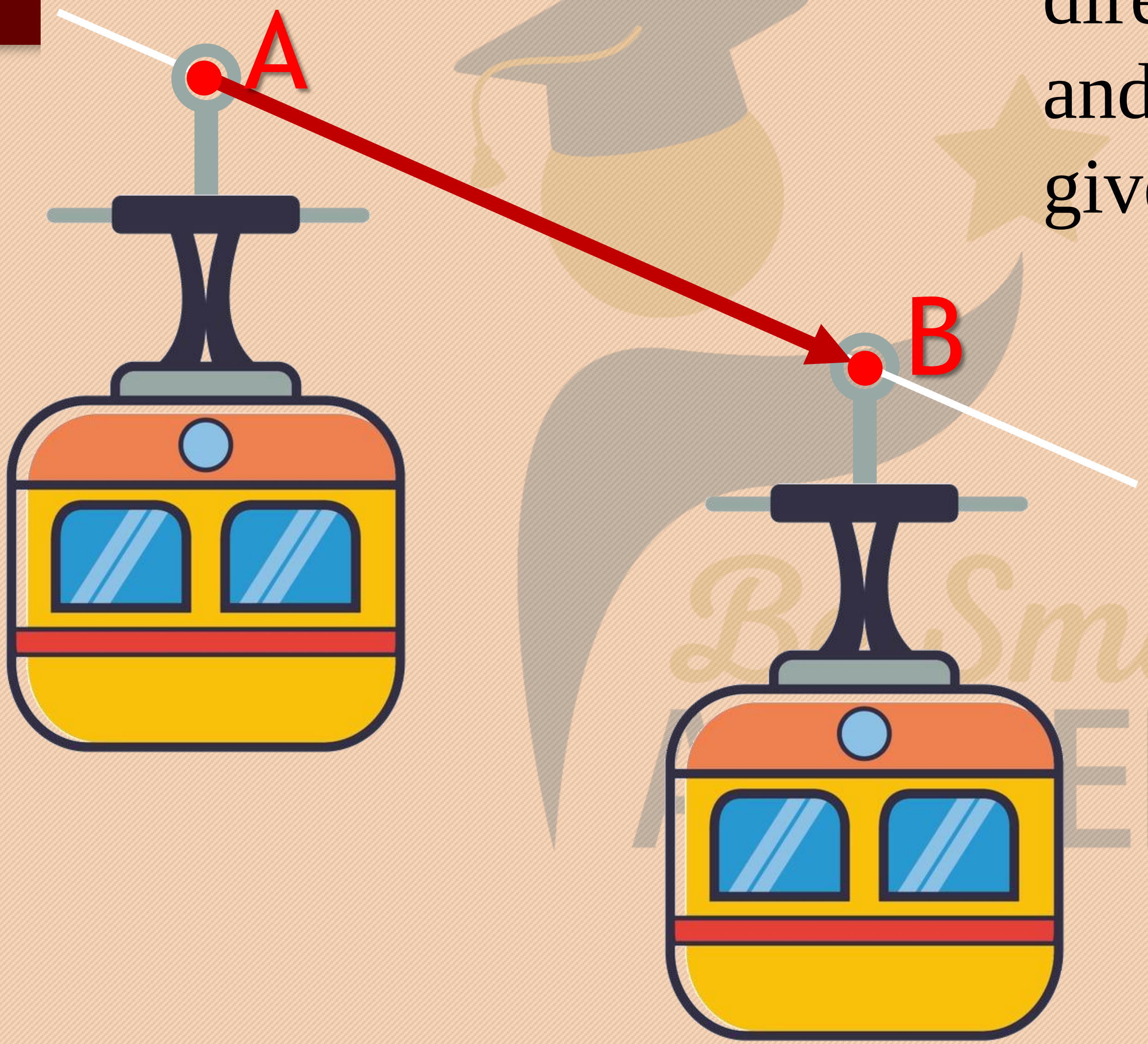


You need to know how much the distance to travel, but you need the direction that help you to travel.

Distance + Direction

01

Introduction



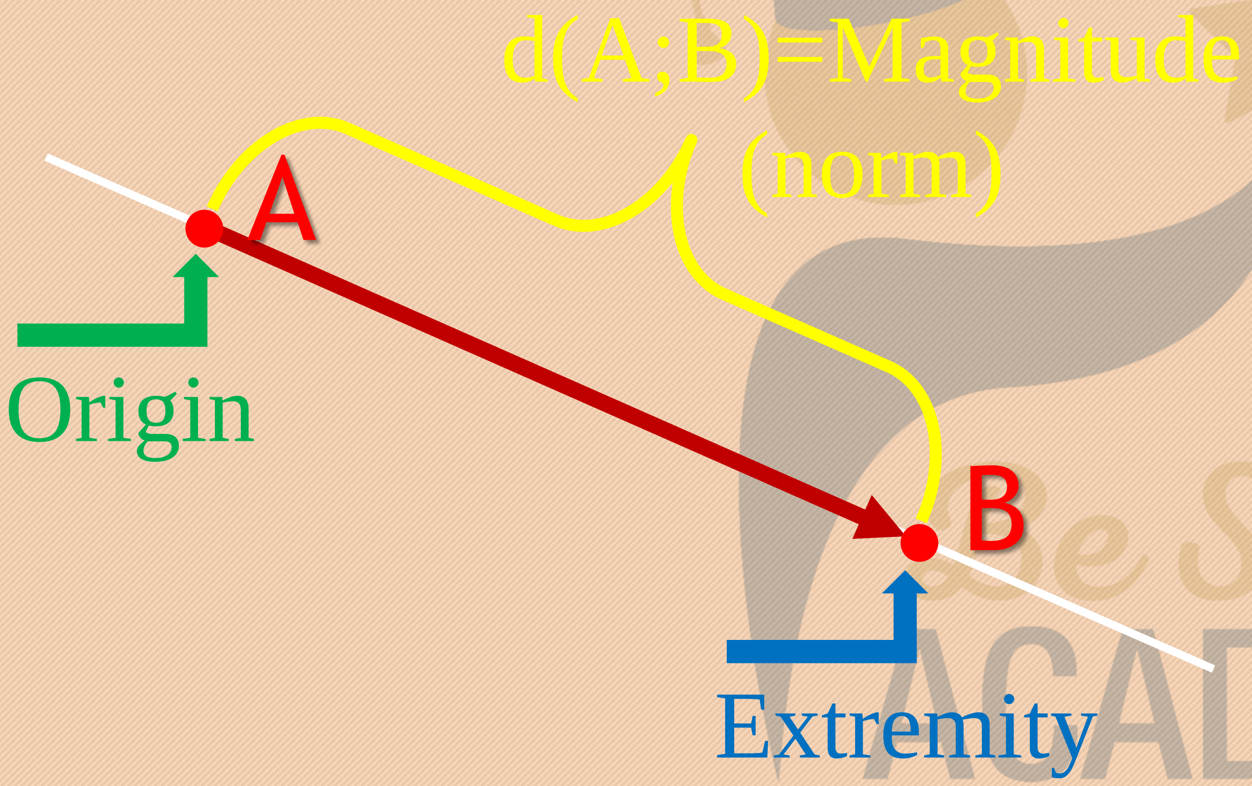
B is the translate of A in a given direction (along (AB)) and sense (from A to B) for a given distance (AB=d(A;B)).

We can define this translation by the vector $\overrightarrow{AB}$ (oriented segment) where:

A is the origin,

B is the extremity.

02 Definition



Vector $\overrightarrow{AB}$

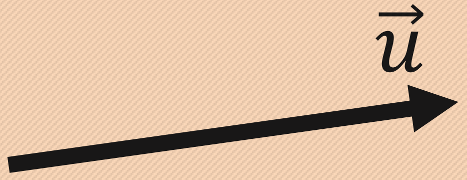
- Direction: (AB)
- Sense: A → B
- Magnitude or norm:
  d(A; B) = AB

Remarks:

- A vector cannot be expressed by a number.

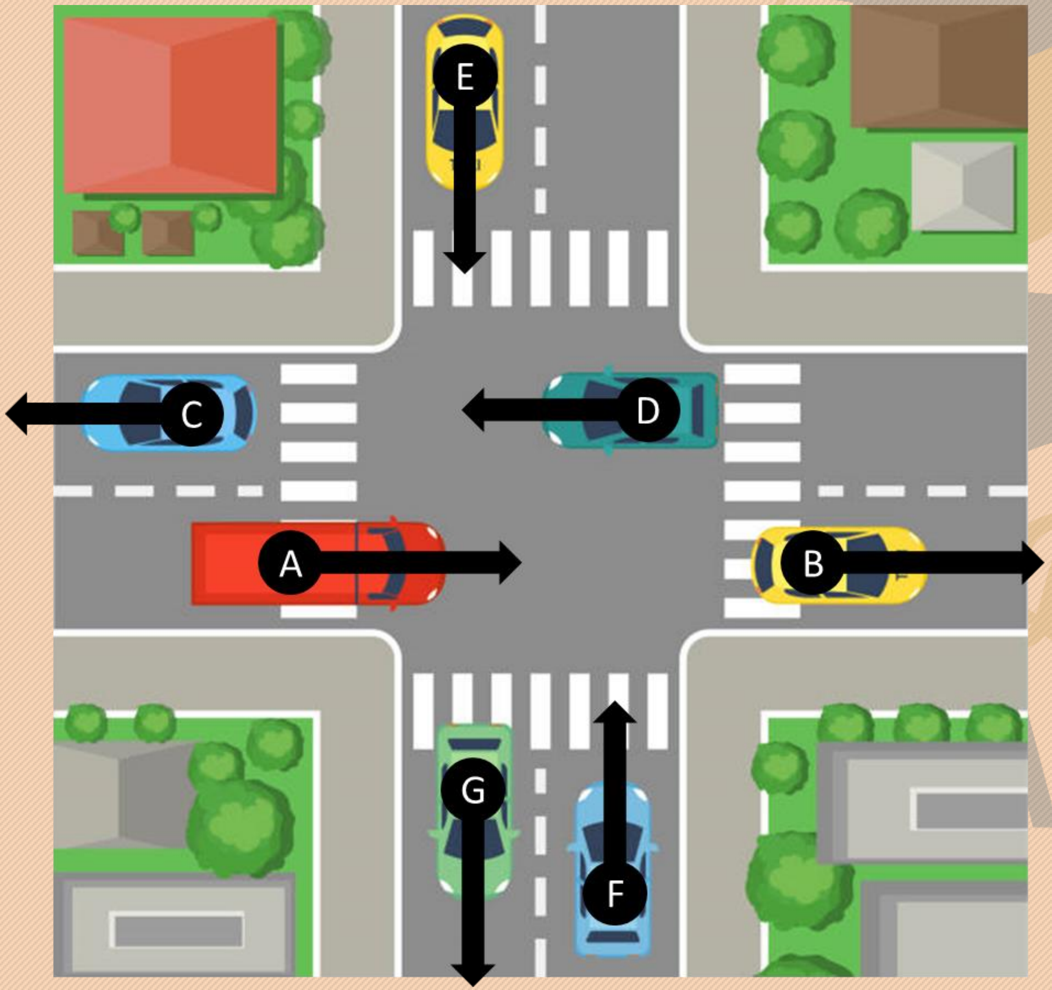
We cannot write $\overrightarrow{AB} = 2$ we write $AB = 2$

- A vector can be named using small letters like $\vec{u}$.



- $\overrightarrow{AB} \neq \overrightarrow{BA}$ (because the sense is different)

Remarks:

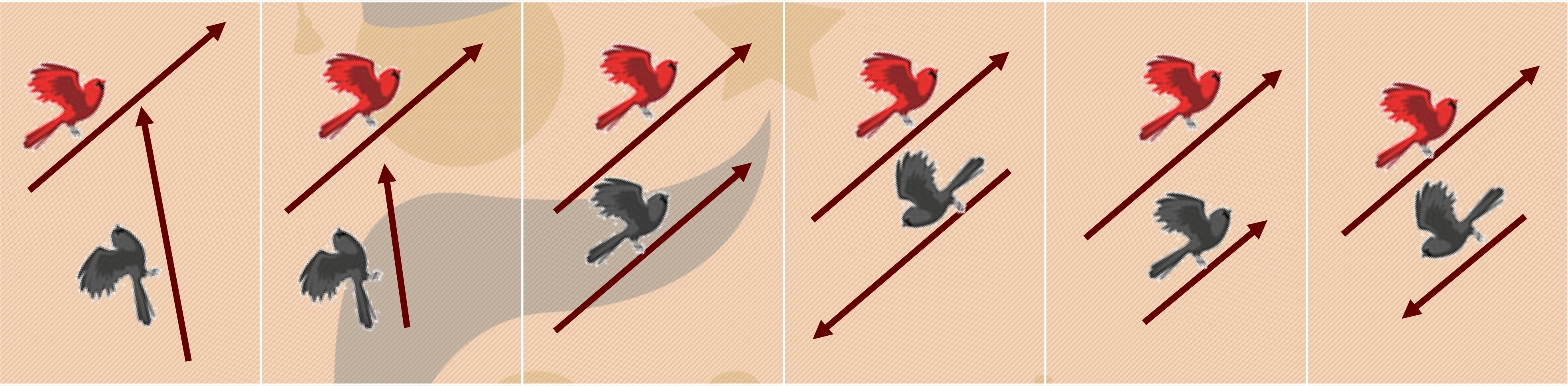


- The direction(road) of cars A and B is parallel to that of cars C and D, so they have same direction.

- In the case of same direction we can determine their sense with respect to each other.

Example:
Car C has a sense opposite to car A but same as car D.
We cannot determine the sense of car E with respect to car C because they have different directions.

03 Different positions of two vectors:

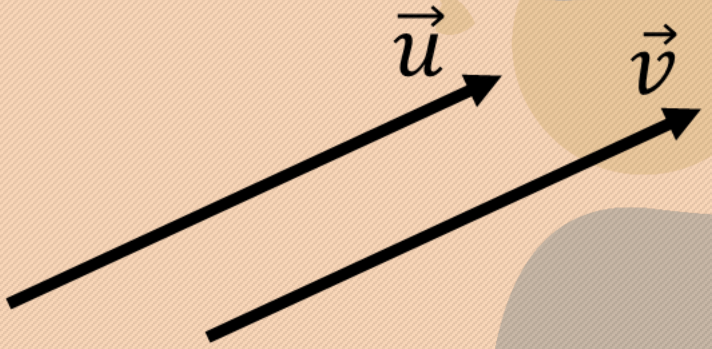


			equal vectors	opposite vectors		
Same direction	**No**	**No**	**Yes**	**Yes**	**Yes**	**Yes**
Same sense	**No**	**No**	**Yes**	**No**	**Yes**	**No**
Same norm	**Yes**	**No**	**Yes**	**Yes**	**No**	**No**

03

Different positions of two vectors:

Equal(equipollent) vectors:



Vectors having:

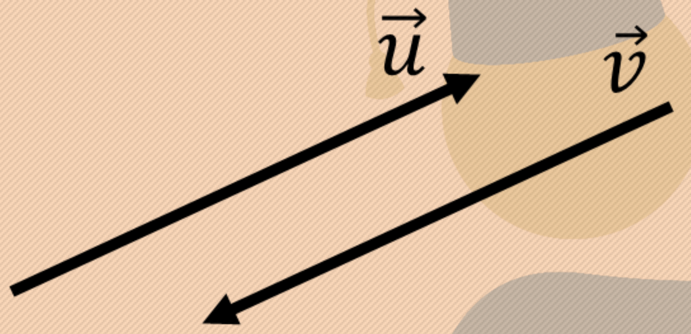
- same direction
- Same sense
- Same norm

Vector relation:

$$\vec{u} = \vec{v}$$

03 Different positions of two vectors:

Opposite vectors:



Vectors having:

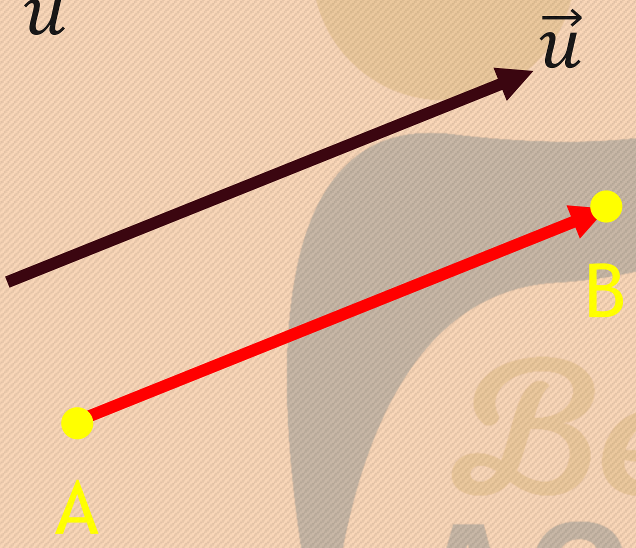
- same direction
- Opposite sense
- Same norm

Vector relation:

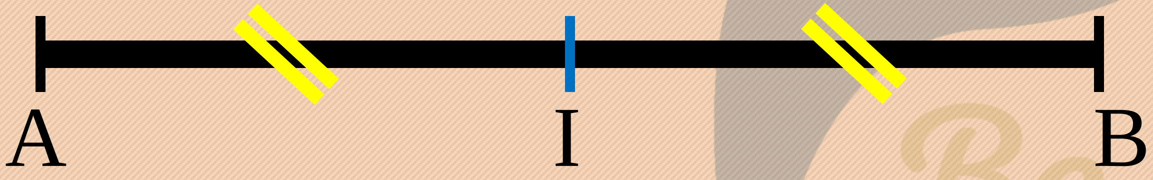
$$\vec{u} = -\vec{v}$$

Remarks:

B is the translate of A by a translation of vector $\vec{u}$: $\overrightarrow{AB} = \vec{u}$



Remarks:

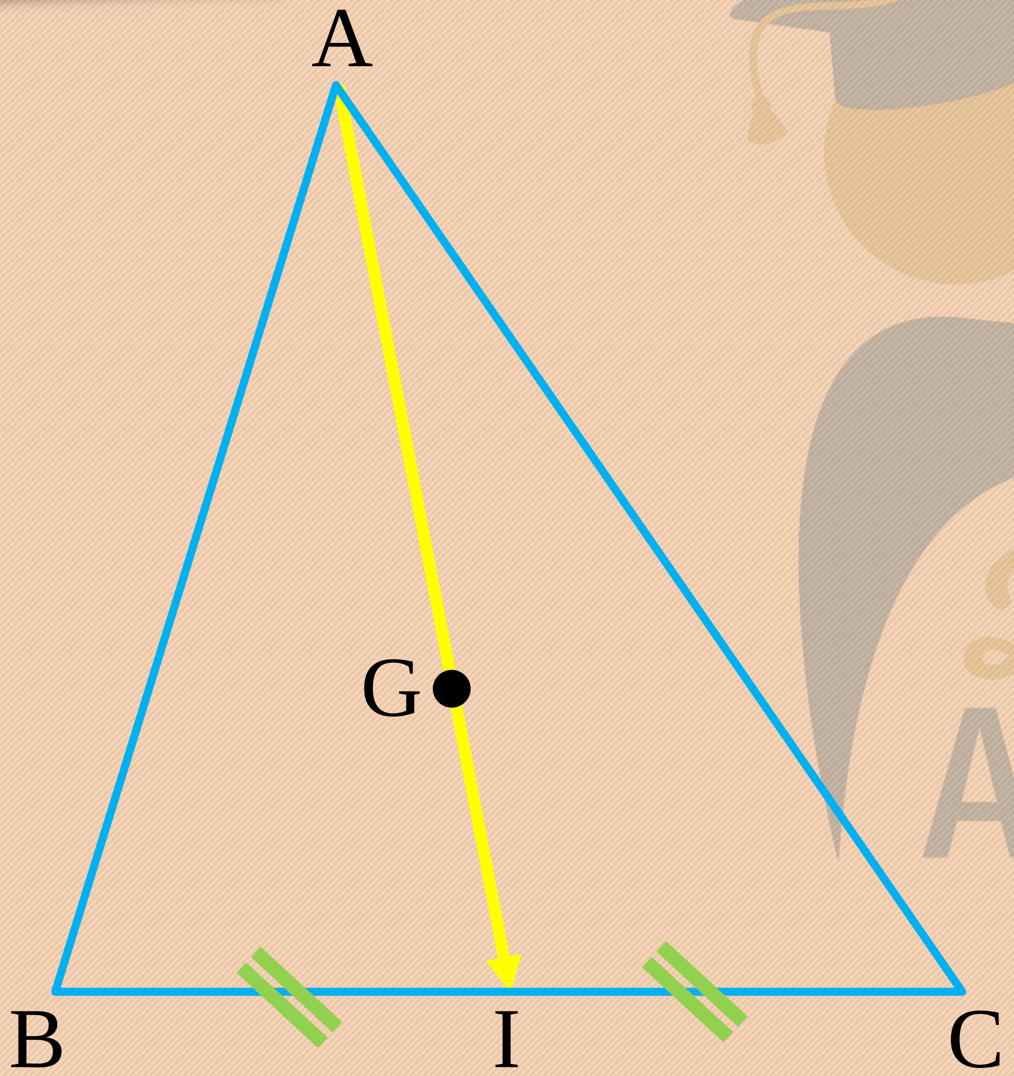


I is the midpoint of [AB]:

$$\overrightarrow{AI} = \overrightarrow{IB} = \frac{\overrightarrow{AB}}{2}$$

$$\overrightarrow{IA} = -\overrightarrow{IB}$$

Remarks:



G is the centroid of the triangle ABC.

$$\overrightarrow{AG} = \frac{2}{3}\overrightarrow{AI} \quad ; \quad \overrightarrow{AI} = \frac{3}{2}\overrightarrow{AG}$$

$$\overrightarrow{GI} = \frac{1}{3}\overrightarrow{AI} \quad ; \quad \overrightarrow{AI} = 3\overrightarrow{GI}$$

$$\overrightarrow{AG} = 2\overrightarrow{GI} \quad ; \quad \overrightarrow{GI} = \frac{1}{2}\overrightarrow{AG}$$

04 Parallelogram

$\overrightarrow{AB} = \overrightarrow{DC}$

$\overrightarrow{DA} = \overrightarrow{CB}$

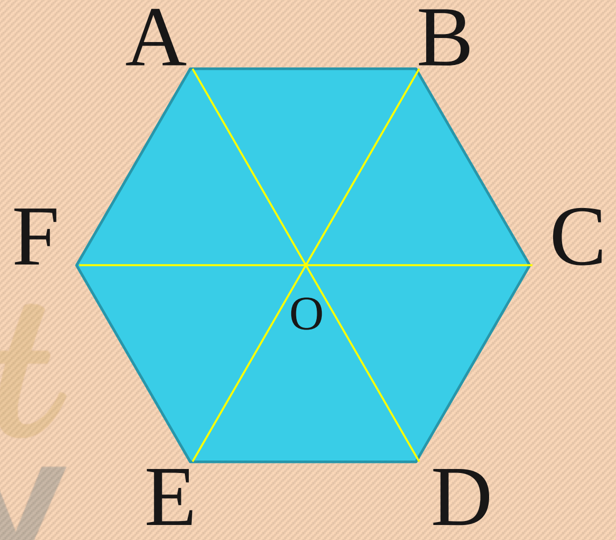
ABCD is a parallelogram

$\overrightarrow{AD} = \overrightarrow{BC}$

$\overrightarrow{BA} = \overrightarrow{CD}$

Application: 01

The following figure shows a regular hexagon.
State 5 equal vectors.



$\overrightarrow{AB} = \overrightarrow{ED}$

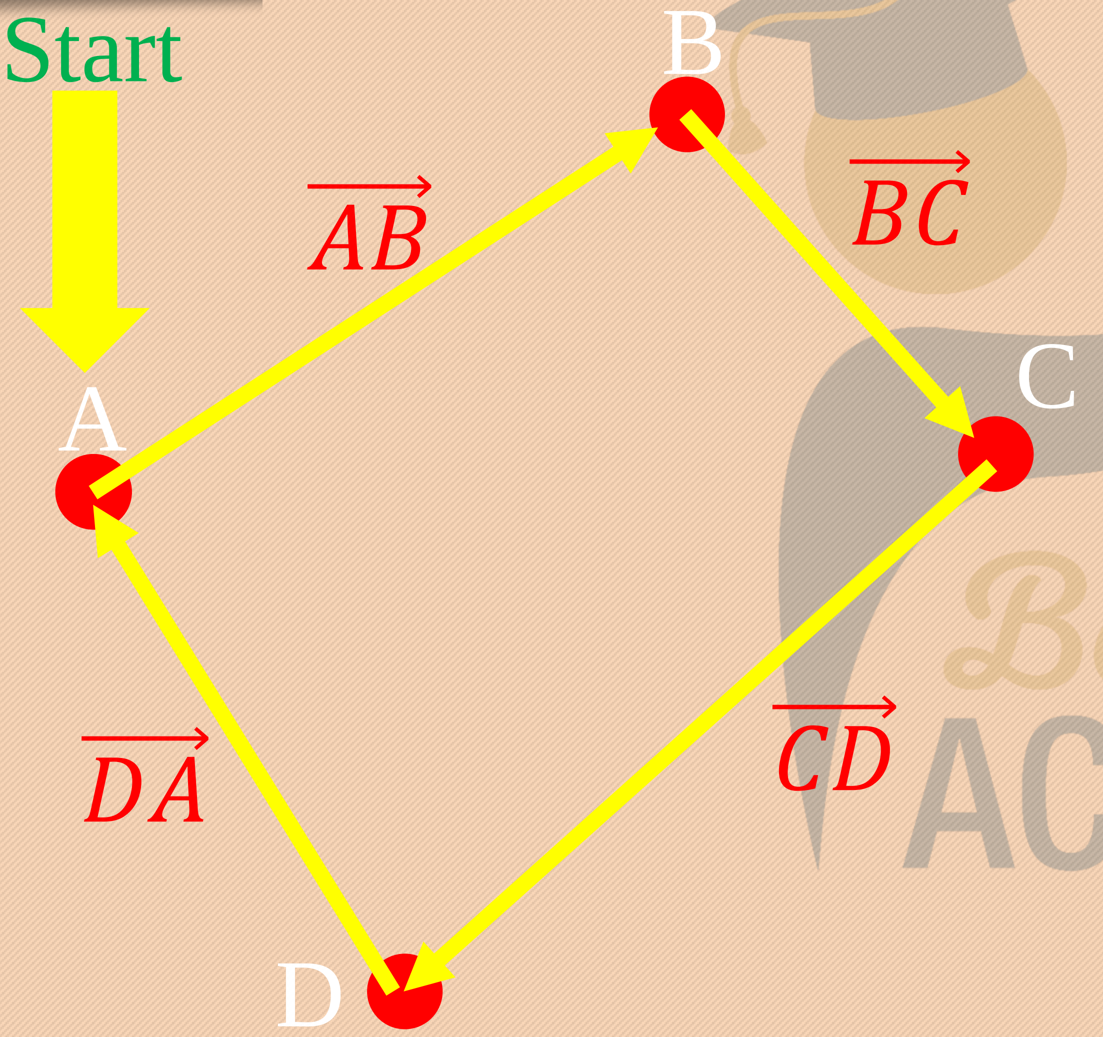
$\overrightarrow{BC} = \overrightarrow{FE}$

$\overrightarrow{CD} = \overrightarrow{AF}$

$\overrightarrow{FE} = \overrightarrow{BC}$

$\overrightarrow{OE} = \overrightarrow{BO}$

05 Zero vector



Origin: A

Extremity: A

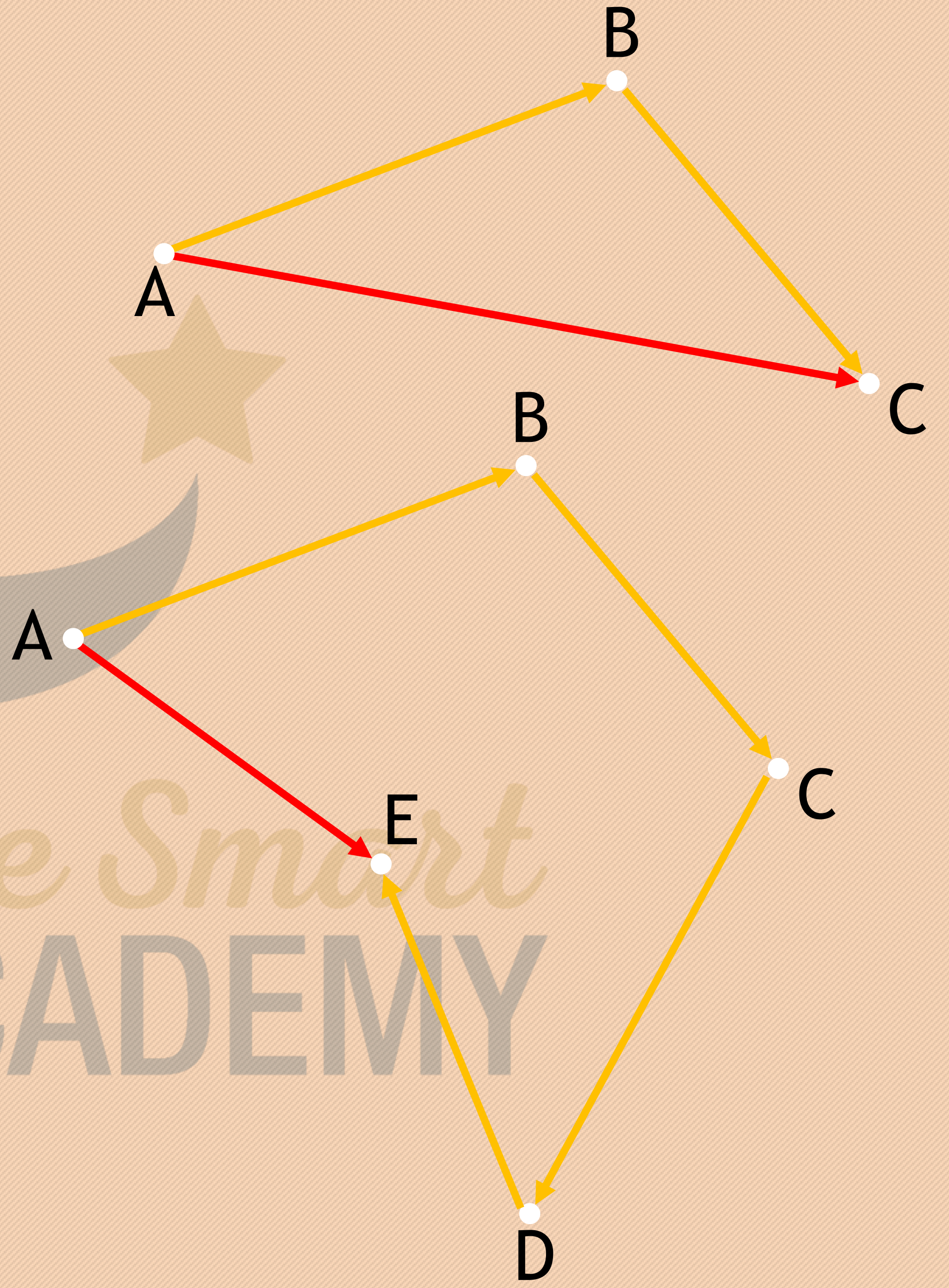
$\overrightarrow{AA} = \vec{0}$ Zero vector or null vector

$$\vec{u} + \vec{0} = \vec{u}$$

06 Chasles's rule

- $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

- $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} = \overrightarrow{AE}$



06

Chasles's rule

Decomposition of a vector using Chasles's rule:

$$\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB}$$

C

We can decompose the vector $\overrightarrow{AB}$ into more than two vectors by inserting more than 1 point between the origin and the extremity following the Chasles' rule.

$$\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EB}$$

Application: 02

Simplify using Chasles' relation.

a) $\vec{u} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$

$$\vec{u} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{AC} + \overrightarrow{CA} = \overrightarrow{AA} = \vec{0}$$

b) $\vec{v} = \overrightarrow{AB} - \overrightarrow{AC} + \overrightarrow{BC} - \overrightarrow{BA}$

$$\vec{v} = \overrightarrow{AB} - \overrightarrow{AC} + \overrightarrow{BC} - \overrightarrow{BA} = \overrightarrow{AB} + \overrightarrow{CA} + \overrightarrow{BC} + \overrightarrow{AB}$$

$$= 2\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 2\,\overrightarrow{AB} + \overrightarrow{BA} = 2\,\overrightarrow{AB} - \overrightarrow{AB} = \overrightarrow{AB}$$

Application: 03

ABCD is a parallelogram.

Show that: $\overrightarrow{MA} - \overrightarrow{MB} + \overrightarrow{MC} - \overrightarrow{MD} = \vec{0}$

$\overrightarrow{MA} - \overrightarrow{MB} + \overrightarrow{MC} - \overrightarrow{MD} =$

$\overrightarrow{MA} - \left(\overrightarrow{MA} + \overrightarrow{AB}\right) + \overrightarrow{MA} + \overrightarrow{AC} - \left(\overrightarrow{MA} + \overrightarrow{AD}\right) =$

$\overrightarrow{MA} - \overrightarrow{MA} - \overrightarrow{AB} + \overrightarrow{MA} + \overrightarrow{AC} - \overrightarrow{MA} - \overrightarrow{AD} =$

$-\overrightarrow{AB} + \overrightarrow{AC} - \overrightarrow{AD} =$

$\overrightarrow{BA} + \overrightarrow{AC} + \overrightarrow{DA} = \overrightarrow{BC} + \overrightarrow{DA}$ but $\overrightarrow{BC} = \overrightarrow{AD}$

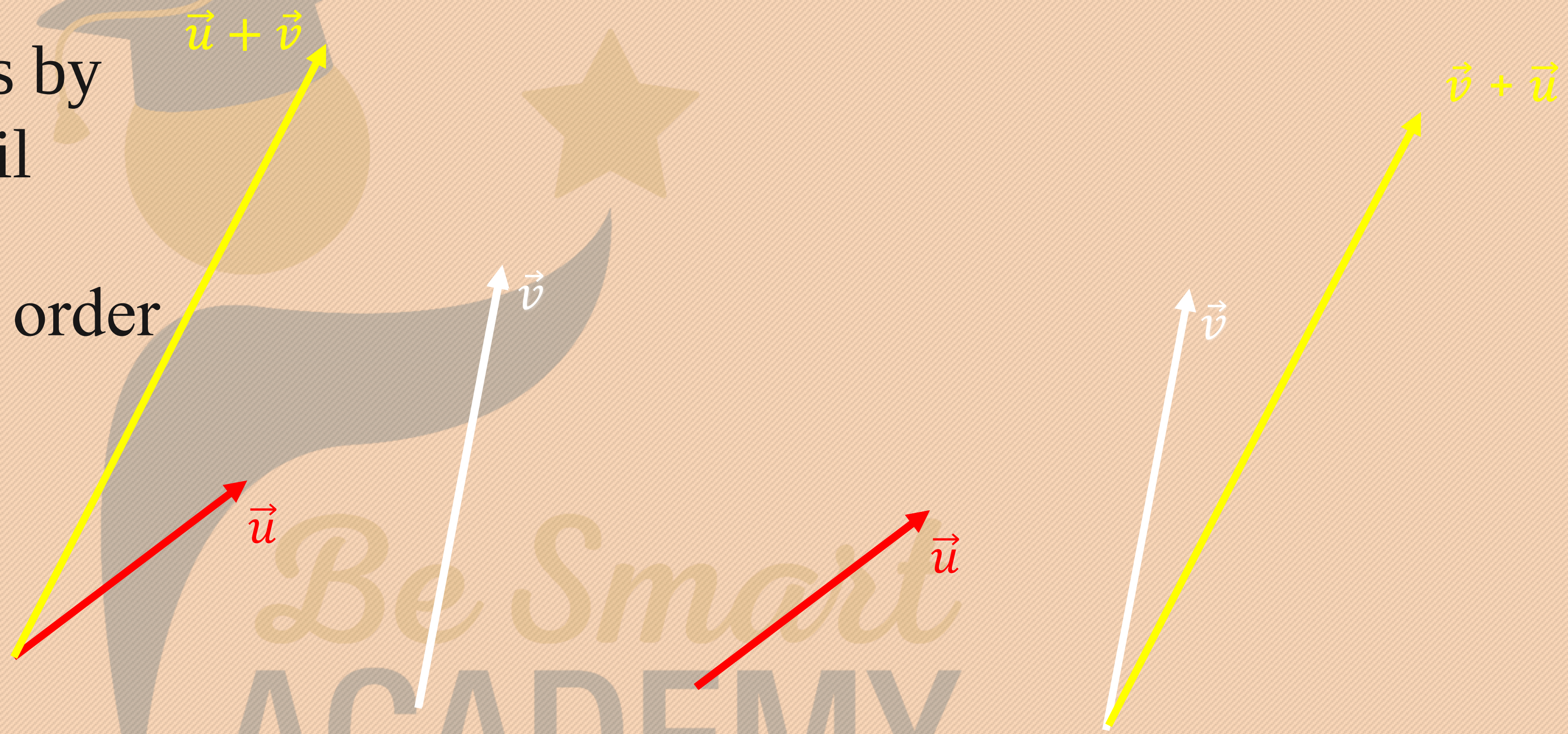
$= \overrightarrow{AD} + \overrightarrow{DA} = \overrightarrow{AA} = \vec{0}$

06

Addition of two vectors.

We can add two vectors by joining them head to tail

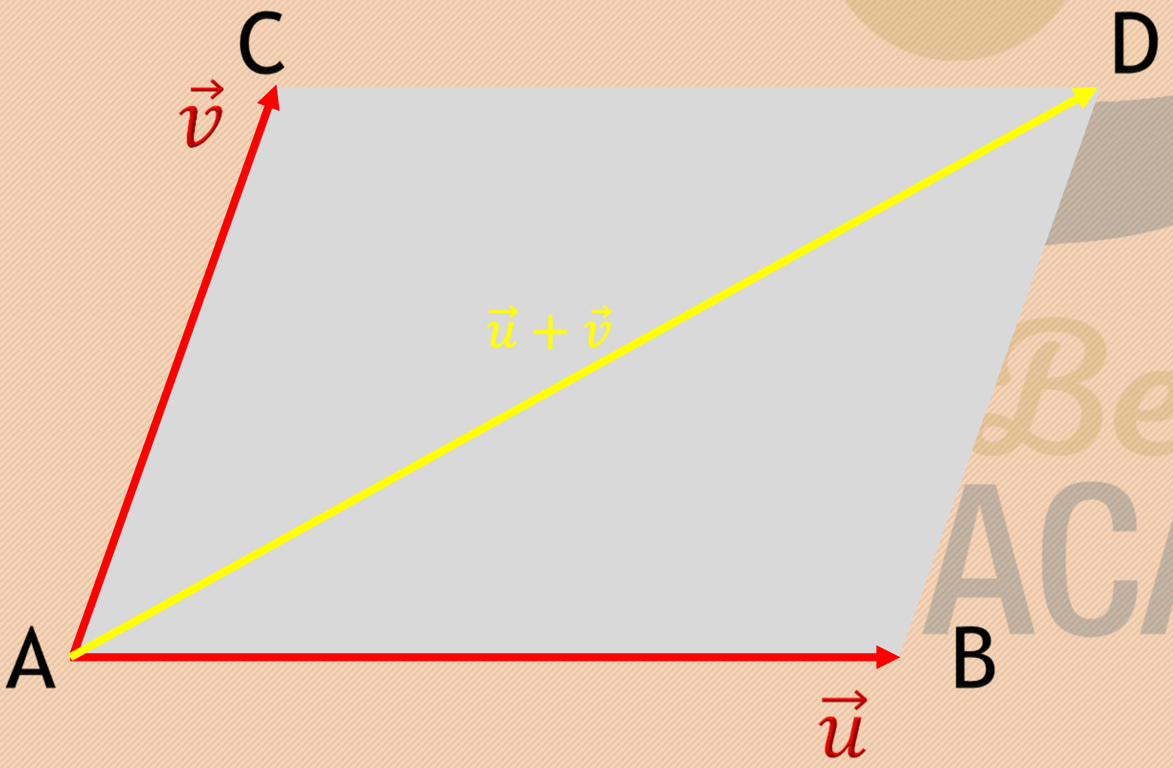
It doesn't matter which order we add them, we get the same result.



06 Addition of two vectors.

Parallelogram rule

$$\vec{u} + \vec{v} = \overrightarrow{AB} + \overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BD} = \overrightarrow{AD}$$



Parallelogram rule:
$\overrightarrow{AB} + \overrightarrow{AC} = \overrightarrow{AD}$ where D is the fourth vertex of the parallelogram ABDC.

Conversely :
If $\overrightarrow{AB} + \overrightarrow{AC} = \overrightarrow{AD}$ then, ABDC is a parallelogram.

07

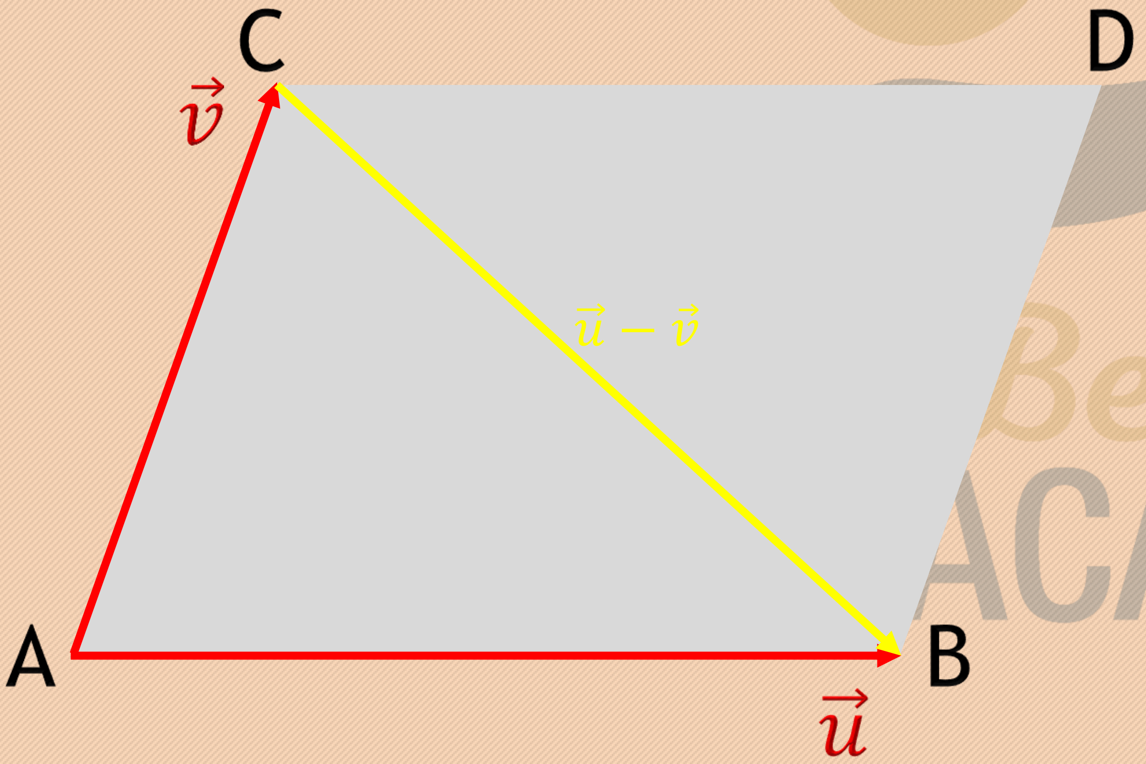
Subtraction of two vectors.

First find $-\vec{v}$ and then add it to $\vec{u}$ (head to tail)

$$\vec{u}-\vec{v} \neq \vec{v}-\vec{u}$$

07 Subtraction of two vectors.

$$\vec{u} - \vec{v} = \overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{CA} = \overrightarrow{CA} + \overrightarrow{AB} = \overrightarrow{CB}$$



$$\overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{CB}$$

10 Vector and system

$A(x_A; y_A)$ and $B(x_B; y_B)$.

$$\overrightarrow{AB}\left|\begin{array}{l} x_{\overrightarrow{AB}} = x_B - x_A \\ y_{\overrightarrow{AB}} = y_B - y_A \end{array}\right.$$

Example:

A(-3;3) and B(1;-1)

$$\overrightarrow{AB}\left|\begin{array}{l} x_{\overrightarrow{AB}} = x_B - x_A = 1 - (-3) = 4 \\ y_{\overrightarrow{AB}} = y_B - y_A = -1 - 3 = -4 \end{array}\right.$$

So $\overrightarrow{AB}(4; -4)$